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AN ITERATIVE APPROACH FOR THE CORRECTION OF ITERATIVE ERRORS

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by

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ABSTRACT

The errors most likely to occur in a high-speed multiplier are called the iterative errors. An arithmetic coding technique for the correction of such error patterns is proposed. We present a class of codes and show its error correcting ability. The unique feature of this code is an iterative decoding method.

I INTRODUCTION

The high-speed multiplier schemes such as the one proposed by MacSorley [1] have been well investigated and implemented in many computers. In such a multiplier scheme, the multiplier is divided into blocks of two (or more) bits each and each block is multiplied to the multiplicant to form partial products. The partial products are then appropriately shifted and added in a multi-input parallel adder with minimum carry provisions. The expected error pattern is quite different from either the multiple independent errors or the burst errors. These errors are shown to be iterative in nature and of the following special form. We let m = the length of a block in bits, r = the number of blocks, and let E be a single iterative error.

__Definition 1__ $E = \pm 2^k \sum_{i=0}^{r-1} e_i 2^{mi}$, where $0 \le k < m$ and $e_i = 0$ or 1 for all i.

A large class of arithmetic codes for the correction of such errors has been developed by Chien and Hong [2,3]. It has been shown that this class of codes has an easy implementation scheme and a nearly optimal rate. We propose a different class of arithmetic codes here, which is based on the concept of an iterative decoding method for the iterative errors.

Arithmetic codes are designed to detect or correct errors in digital computations. One such error may change many output digits by propagations. Single error correcting codes are summarized in Peterson [4], and multiple independent error correcting codes have been studied by Barrows [5], Mandelbaum [6], Chang and Tsao-Wu [7] and Chien, Hong, and Preparata [8,9]. Burst error correcting arithmetic codes have been investigated by Stein [10], Chien [11], and Mandelbaum [12].

Arithmetic codes are of the form AN, where A is a fixed integer called the generator. N is an integer in the interval (0, B-1), and B is the number of code words. If the code length is n, B is the smallest integer such that $AB>2^n$. In the binary case, A is obviously an odd number. The error correcting capability of ordinary AN codes depends on the minimum distance of the code, which in turn depends on the generator A. A corrupted signal (correct signal plus error) modulo A is called the syndrome of the error which is the same as the error modulo A. Syndrome of an error, usually denoted as S, then leads to the correct decision of the error through the decoding algorithms.

II. DERIVATION OF THE CODE

It follows from the definition that to correct the error one must correctly determine the polarity of the error, the position of the error (4) and the distribution of the erroneous digits, i.e., the set of e_i's. The class of codes dealt with in this work is for the cases when the number of blocks, r, is two to the some power, i.e., $r = 2^{t_1}$ for some $t_1 \geq 1$. Note that the length of the code is $m2^{t_1}$ and $2^{mr}-1$ is now divisible by $2^{m2^i} \pm 1$ for all $0 \leq i \leq t_1-1$.

The Polarity of Error

Let t_0 be some integer less than t_1. Consider the positive error modulo $2^{m2^{t_0}}-1$. Clearly

$$E = E' = +2^k \sum_{i=0}^{2^{t_1}-1} e_i 2^{mi} \equiv 2^k \sum_{i=0}^{2^{t_0}-1} f_i 2^{mi} \bmod 2^{m2^{t_0}}-1$$

where $0 \le f_i \le 2^{t_1 - t_0}$ for all $0 \le i \le 2^{t_0} - 1$. Thus, each f_i can have at the most $(t_1 - t_0)$ 1's in its binary form. If $t_1 - t_0 < \frac{1}{2}m$, the whole residue must have less than $(2^{t_0 - 1} m)$ 1's.

Lemma 1 Given $t_0 > t_1 - \frac{1}{2}m$ (1)

$S \equiv E \bmod 2^{m2^{t_0}} - 1$ has less than $(2^{t_0 - 1} m)$ 1's if and only if the polarity of error is positive.

Proof We must show that when the polarity is negative, S has greater than $(2^{t_0 - 1} m)$ 1's. Let $E = -E'$ and $S' \equiv E' \bmod 2^{m2^{t_0}} - 1$. We know that S' has less than $(2^{t_0 - 1} m)$ 1's. Therefore,

$$S = 2^{m2^{t_0}} - 1 - S'$$

and the number of ones in S is greater than $m2^{t_0} - (m2^{t_0 - 1}) = m2^{t_0 - 1}$. We mention here that $S = 0$ only if $E = 0$, i.e., no error.

Q.E.D.

Intermediate Error Pattern

Using the same notation as $E = \pm E'$, or

$$E' = 2^k \sum_{i=0}^{2^{t_1} - 1} e_i 2^{mi} \qquad (2)$$

we now define an intermediate error pattern as

$$\mathcal{E}_j \equiv E' \bmod 2^{m2^j} - 1 \qquad (3)$$

for all $t_0 \leq j \leq t_1$. Clearly, $\mathcal{E}_{t_1} = E'$; and from Eq. (2) we have

$$\mathcal{E}_j = 2^k \sum_{i=0}^{2^j-1} a_i 2^{mi} \tag{4}$$

where $0 \leq a_i \leq 2^{t_1-j}$ and $0 \leq k < m$. Also, note that $\mathcal{E}_j \equiv \mathcal{E}_{j+1} \bmod 2^{m2^j}-1$ for all $j < t_1$.

Consider an intermediate error pattern, $\mathcal{E}_j$, given in an ordinary binary form. Each a_i becomes a burst* of length at most t_1-j with at least $m-(t_1-j)$ 0's in between. These bursts can be uniquely recognized if $t_1-j < \frac{1}{2} m$, i.e., if $j > t_1 - \frac{1}{2} m$. Let k_j be the maximum integer such that $2^{k_j} a_i \leq 2^{t_1-j}$ for all i, for the given $\mathcal{E}_j$ of Eq. (4). Clearly, $k_j \geq 0$. Now denote by $\underline{\mathcal{E}}_j$ the following equation which is numerically the same as $\mathcal{E}_j$.

$$\underline{\mathcal{E}}_j = 2^{(k-k_j)} \sum_{i=0}^{2^j-1} (a_i 2^{k_j}) 2^{mi} \tag{5}$$

<u>Lemma 2</u> If $t_0 > t_1 - \frac{1}{2} m$, $\underline{\mathcal{E}}_j$ can be uniquely determined from the binary pattern of $\mathcal{E}_j$, for all $t_0 \leq j \leq t_1 - 1$.

<u>Proof</u> $t_0 > t_1 - \frac{1}{2} m$ implies $j > t_1 - \frac{1}{2} m$ for all given j's. Thus, the bursts of a_i's are uniquely recognized for all j. Now mark the position of $\lceil \frac{m}{2} \rceil$** th bit after the longest burst and each m th bit positions thereafter,

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*The term, "burst", denotes a binary pattern beginning and ending with 1's. A single 1 is considered as a burst of length one. A cyclic connection between $m2^j$ th bit and the first bit is assumed.

**$\lceil x \rceil$ denotes the least integer greater than or equal to x.

cyclically around the entire length of 2^{m2^j} bits. These marks fall among the 0's separating the bursts. Let the position of the smallest marked bit be k', we have

$$2^{k'} \sum_{i=0}^{2^j-1} (a_i 2^{k-k'}) \cdot 2^{mi} \equiv \mathcal{E}_j \bmod 2^{m2^j}-1$$

Now, change k' until $(a_i 2^{k-k'}) \leq 2^{t_1-j}$ for all i, for the first time. By the definition of k_j, $k' = k-k_j$ and $k-k' = k_j$ for Eq. (5). We mention here that any time $(a_i 2^{k_j})$ becomes an odd number, $k_j = 0$ and the position of the error, $k = (k-k_j)$.

Q.E.D.

Suppose a binary pattern of $\mathcal{E}_j$ is given and $(k-k_j)$ and $(a_i 2^{k_j})$'s are all decided according to lemma 2. We let

$$\mathcal{E}'_{j+1} = 2^{(k-k_j)} \sum_{i=0}^{2^{j+1}-1} b'_i \, 2^{mi} \tag{6}$$

where $0 \leq b'_i \leq 2^{t_1-j-1}$ and $b'_i + b'_{i+2^j} = (a_i 2^{k_j})$ for all i.

<u>Lemma 3</u> Let $\mathcal{E}_{j+1} = 2^k \sum_{i=0}^{2^{j+1}-1} b_i 2^{mi}$. $0 \leq b_i \leq 2^{t_1-j-k_j}$ for all $0 \leq i \leq 2^{j+1}-1$.

<u>Proof</u> From Eq. (4), we know that $0 \leq b_i \leq 2^{t_i-j-1}$ for all i. Also, from the definition of $\mathcal{E}_j$, $a_i = b_i + b_{i+2^j}$ for all $0 \leq i \leq 2^j-1$. Now, since $(a_i 2^{k_j}) \leq 2^{t_1-j}$, $(b_i + b_{i+2^j})^{k_j} \leq 2^{t_1-j}$. Thus, $b_i \leq 2^{t_1-j-k_j}$ regardless of k_j.

Q.E.D.

The β-Code

Define a class of integers, β_j, as the following. β_j is a prime factor of $2^{m2^j}+1$, such that $x = m2^j$ is the least positive solution for $2^x+1 \equiv 0 \bmod \beta_j$. β_j is said to have order n if

$$\sum_{i=0}^{2^{j+n}-1} e_i 2^{mi} \not\equiv 0 \bmod 2^{m2^j}+1 \text{ implies } \sum_{i=0}^{2^{j+n}-1} e_i 2^{mi} \not\equiv 0 \bmod \beta_j$$

where e_i = 1 or 0 for all i. An equivalent condition is

$$\sum_{i=0}^{2^j-1} a_i 2^{mi} \not\equiv 0 \bmod \beta_j \qquad (7)$$

where $|a_i| \leq 2^{n-1}$ for all i and not all a_i's are 0.

Finding the order of given β_j seems to be a difficult number theory problem. But one can easily find the order by a computer programming. Table 1 shows a short list of β_j's and the orders. It appears that all the β_j's have order at least one.

Table 1. β_j and order.

m	j	β_j	order	m	j	β_j	order
3	1	13	2	6	4	97	2
3	2	241	2	6	8	193	1
5	1	41	2	7	2	29	3
5	2	61681	8	7	2	113	4
6	1	241	8	7	4	1579031	5
6	2	673	2	7	8	5153	1

For a given m and $r = 2^{t_1}$, the β-code is defined under the following assumptions. i) $t_1 > t_0 > t_1 - \frac{1}{2} m$ and $t_0 \geq 0$; ii) there exist β_j's ($t_0 \leq j \leq t_1-2$) of order at least t_1-j+2 and β_{t_1-1} of order 2. When such β_j's exist we define the generator of the β-code as

$$A_\beta = (2^{m2^{t_0}}-1)\beta_{t_0}\,\beta_{t_0+1}\cdots\beta_{t_1-1} \tag{8}$$

We mention here that this generator divides $2^{mr}-1$ and therefore resembles the form of the generators for ordinary multiple error correcting arithmetic codes [5-10].

III. ITERATIVE DECODING

The decoding is done by iteratively determining the intermediate error patterns. We first show how $\mathcal{E}_{j+1}$ is obtained from given $\mathcal{E}_j$ and present the complete decoding algorithm. An example follows for illustration.

Lemma 4 Assume the order of β_j is greater than or equal to t_1-j+2. $\mathcal{E}'_{j+1} \equiv \mathcal{E}_{j+1} \bmod \beta_j$ if and only if $\mathcal{E}_{j+1} = \mathcal{E}'_{j+1}$, for all $t_0 \leq j \leq t_1-1$.

Proof We must show that $\mathcal{E}_{j+1} \equiv \mathcal{E}'_{j+1} \bmod \beta_j$ implies $\mathcal{E}_{j+1} = \mathcal{E}'_{j+1}$. Now,

$$\mathcal{E}_{j+1} = 2^{k-k_j}\sum_{i=0}^{2^{j+1}-1} b_i 2^{k_j}\, 2^{mi} \equiv 2^{k-k_j}\sum_{i=0}^{2^{j+1}-1} b'_i\, 2^{mi} \bmod \beta_j$$

or

$$\sum_{i=0}^{2^{j+1}-1} (b_i 2^{k_j} - b'_i) 2^{mi} \equiv 0 \bmod \beta_j$$

but

$$2^{mi} \equiv -2^{m(i+2^j)} \bmod \beta_j$$

Thus
$$\sum_{i=0}^{2^j-1} \{(b_i - b_{i+2^j})2^{k_j} - (b'_i - b'_{i+2^j})\} 2^{mi} \equiv 0 \bmod \beta_j$$

Since $|(b_i - b_{i+2^j})| \leq 2^{t_1-j-k_j}$ by lemma 3 and $|b'_i - b'_{i+2^j}| \leq 2^{t_1-j-1}$, $|\{(b_i - b_{i+2^j})2^{k_j} - (b'_i - b'_{i+2^j})\}| \leq 2^{t_1-j} + 2^{t_1-j-1} \leq 2^{t_1-j+1}$ for all j. By the definition of β_j,

$$(b_i - b_{i+2^j})2^{k_j} - (b'_i - b'_{i+2^j}) = 0$$

But

$$(b_i + b_{i+2^j})2^{k_j} = a_i 2^{k_j} = (b'_i + b'_{i+2^j})$$

Therefore $b'_i = b_i 2^{k_j}$ for all $0 \leq i \leq 2^{j+1}-1$.

Q.E.D.

<u>Theroem 9</u> The β-codes, when exist, correct all single iterative errors.

<u>Proof</u> Let the initial syndrome be $S_0 \equiv A_\beta N + E \equiv E \bmod A_\beta$.

Step 1) If $h(S \bmod 2^{m2^{t_0}}-1) < m2^{t_0-1}$, the polarity is positive, and otherwise negative. (By lemma 1.) If positive $S_1 = S_0$, and if negative $S_1 = A_\beta - S_0$. In either case $S_1 \equiv E' \bmod A_\beta$.

Step 2) $\mathcal{E}_{t_0} \equiv E' \equiv S_1 \bmod 2^{m2^{t_0}}-1$. However, the $\mathcal{E}_{t_0}$ obtained now is in binary pattern. Iteratively follow the next step for $t_0 \leq j \leq t_1-2$.

Step 3) From the binary $\mathcal{E}_j$, find $\underline{\mathcal{E}}_j$ by lemma 2. Using $S_1 \equiv E_j \bmod \beta_j$, find $\mathcal{E}_{j+1}$ uniquely from $\underline{\mathcal{E}}_j$ by lemma 4.

Step 4) Let $\mathcal{E}_{t_1-1} = 2^{k'}\sum_{i=0}^{2^{t_1-1}-1} a_i 2^{mi}$. i) If $S_1 \equiv 0 \bmod \beta_{t_1-1}$, then $a_i = 0$ or 2 for all i and $k = k'$. ii) If $S_1 \not\equiv 0 \bmod \beta_{t_1-1}$ and $a_i = 0$ or 2 for all i, then $k = k'+1$. iii) If $S_1 \not\equiv 0 \bmod \beta_{t_1-1}$ and $a_i = 0$, 1, or 2 for all i, then $k = k'$

Step 5) Let $\mathcal{E} = \mathcal{E}_{t_1-1} = 2^k \sum_{i=0}^{2^{t_1-1}-1} a_i 2^{k'-k}\, 2^{mi}$

$$\text{Let } \mathcal{E}'_{t_1} = 2^k \sum_{i=0}^{2^{t_1-1}-1} e'_i\, 2^{mi} \tag{9}$$

where $0 \leq e'_i \leq 1$ for all i and $e'_i + e_{i+2^{t_1-1}} = a_i 2^{k'-k}$ for all i.

By the same arguments as lemma 4, $\mathcal{E}'_{t_1} = \mathcal{E}_{t_1} = E$ if and only if $\mathcal{E}'_{t_1} = \mathcal{E}_{t_1} \bmod \beta_{t_1-1}$. Q.E.D.

<u>Example</u> Let m = 6. Table 1 gives $\beta_1 = 241$ with order 8 and $\beta_2 = 673$ with order 2. Let $t_1 = 2$, i.e. r = 8. $t_0 = 1$ satisfies the condition for β-code, thus

$$A_\beta = (2^{12}-1)\ 241\cdot 673$$

the rate of which is approximately 0.4. Suppose the error is $(e_0, e_1, \ldots, e_7)$ = (10110101), k = 3 and of positive polarity. $\mathcal{E}_2 \equiv \overline{E'} \bmod 2^{12}-1$ becomes the following binary pattern with the marks, $\uparrow$. ($[\frac{6}{2}] = 3$)

0	1	2	3	4	5	6	7	8	9	10	11	→ binary positions
0	0	0	0	1	0	0	0	0	1	1	0	→ (11) is the longest burst
	↑first mark						↑second mark					→ marking

$k'=1,\ (a_1 2^{k-k'}) = 8,\ (a_2 2^{k-k'}) = 12$ → applying lemma 2

$(k-k_2) = 3,\ (a_1 2^{k_2}) = 2,\ (a_2 2^{k_2}) = 2$

$\underline{\mathcal{E}}_1 = 2^3(2\cdot 2^0 + 3\cdot 2^6)$ → Eq. (5)

$0\le b'_i\le 2,\ b'_0+b'_2 = 2,\ b'_1+b'_3 = 3$ → for Eq. (6)

$\underline{\mathcal{E}}_2 = 2^3(1\cdot 2^{6\cdot 0}+1\cdot 2^{6\cdot 1}+1\cdot 2^{6\cdot 2}+2\cdot 2^{6\cdot 3})$ → by lemma 4

$S_1 \equiv E_1 \not\equiv 0 \bmod 673$, thus $\underline{k = 3}$ → by step 4.

$e'_i\ge 0;\ e'_0+e'_4=e'_1+e'_5=e'_2+e'_6=e'_3=e'_7=1$ → Eq. (9), Step 5

$\mathcal{E}'_3 \equiv S_1 \bmod 673$ when $(e'_0,e'_1,\ldots,e'_7)=(10110101)$. Thus decoded.

IV. CONCLUSION

The β-codes are based on an interesting decoding method, namely, an iterative decoding for the iterative errors. From the syndrome of an iterative error, intermediate error patterns are iteratively decoded, each time doubling the length of the pattern. Although some searching and matching operations are necessary at each step, the unusual feature of this decoding technique may be desired for some applications.

The β-codes of high rate do not seem to exist for small m's. However, for large m, it is very probable that such β_j's exist. The rate of the β-code is generally less than the rate of the codes described in [3]. Again, for large m, the rate of β-code is likely to improve.

The decoder design, a theory or a simple method to find the order of β_i and a proof of existence of β-codes for large m are interesting problems for further research. Also, the iterative decoding concept may find a useful application in the polynomial codes.

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13. ABSTRACT

The errors most likely to occur in a high-speed multiplier are called the iterative errors. An arithmetic coding technique for correction of such error patterns is proposed. We present a class of codes and show its error correcting ability. The unique feature of this code is an iterative decoding method.

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